

FYJC - MATHEMATICS & STATISTICS

HIGHLIGHTS

- ✓ *Solution to all questions*
- ✓ *solutions are put in way the student is expected to reproduce in the exam*
- ✓ *taught in the class room the same way as the solution are put up here . That makes the student to easily go through the solution & prepare him/herself when he/she sits back to revise and recall the topic at any given point of time .*
- ✓ *lastly, if student due to some unavoidable reasons , has missed the lecture , will not have to run here and there to update his/her notes .*
- ✓ *however class room lectures are must for easy passage of understanding & learning the minuest details of the given topic*

PAPER - II

PERMUTATIONS

- 1.- *Fundamental principle of Counting*Pg 05
- 2.- *Arrangement of people* ...Pg 10
- 3.- *Arrangement of Letters*Pg 16
- 4.- *Arrangement of digits* Pg 23
- 5.- *Sums on ${}^n P_r$* Pg 28

FUNDAMENTAL PRINCIPLE OF COUNTING

01. There are 8 routes from city A to city B , 10 routes from city B to city C . Find the number of different ways for a person to travel from city A to city C via city B . **ans : 80 .**
02. a room has six doors . In how many ways can a man enter the room through one door and come out through a different door . **ans : 30**
03. There are 10 trains plying between Calcutta and Delhi . The number of ways in which a person can go from Calcutta to Delhi and return by a different train **ans : 90**
04. in how many ways can 3 prizes be distributed among 4 boys when no boy gets more than one prize . **ans : 24**
05. in how many ways can 4 prizes be distributed among 3 boys if a boy can get any number of prizes **ans : 81**
06. in how many ways can 3 students be associated with 4 chartered accountants if each chartered accountant can have at most 1 student . **ans : 24**
07. A letter lock contains 3 rings , each ring contains 5 different letters . Determine the maximum number of false trials that can be made before lock is opened **ans : 124**
08. For a set of 5 true/False questions no student has written all correct answers and no two students have given the same sequence of answers . What is the maximum number of students in the class for this to be possible **ans : 31**
09. Given 4 flags of different color , how many different signals can be generated if a signal requires to use 2 flags one below the other . **ans : 12**
10. there are 6 items in column A and 6 items in column B . A student is asked to match each item in column A with an item in column B . How many possible , correct , incorrect answers are there to this question **ans : 720**
11. How many 3 digit numbers can be formed from the digits 0, 2, 4 , 5, 7 if the repetition of digits is not allowed. **ans : 48**

12. How many 3 digit even numbers can be formed from the digits 1 , 2 , 3 , 4 , 5 if the digits can be repeated **ans : 50**

13. How many numbers between 100 and 1000 are such that exactly one of the digits is 6. **ans : 225**

PERMUTATION - QSET 1

01. 3 Asiatics ; 2 Europeans and 1 American stand in a line for a photograph . Find the number of arrangements so that 3 Asiatics are together and so are 2 Europeans . **ans : 72**

02. There are 7 students of whom 2 are Americans , 2 Russians and 3 Indians . They have to stand in a line so that 2 Americans are always together and 3 Indians are always together . In how many ways can this be done **ans: 288**

03. 6 boys and 2 girls are to be stand in a line for a photograph . Find the number of arrangements such that

a) two girls sit next to each other **ans : 10080**

b) two girls occupy the end seats **ans : 1440**

04. there are 4 books on Physics and 2 on Maths . Find the number of ways in which the books can be arranged so that books on Mathematics are not together **ans : 480**

05. Five persons are to be seated in a row . Find the number of seating arrangements

a) if two persons X and Y are always together **ans : 48**

b) if two person X and Y never sit together **ans : 72**

c) if X and Y occupy chairs at two ends **ans : 12**

06. In how many ways can 6 papers of which 2 are math can be arranged such that

a) two math paper are consecutive **ans : 240**

b) the two math paper are not consecutive **ans : 480**

07. A family of 3 sisters and 5 brothers is to be arranged for a photograph in one row . In how many ways can they be seated if

a) all the sisters sit together **ans : 4320**

b) no two sisters sit together **ans : 14400**

08. 7 persons sit in a row . Find the total number of seating arrangements if
- a) 3 persons A , B ,C sit together in particular order **ans : 120**
 - b) A , B and C sit together in any order **ans : 720**
 - c) A and B occupy the end seats **ans : 240**
 - d) C always occupies the end seats **ans: 1440**
09. the college day committee consists of a Principal , four professors and two students . They are seated in a row for a photograph so that the Principal is in the center and the two students occupy the chairs at the ends . how many different photographs can be taken **ans : 48**
10. 6 professors , 1 lady and 4 students are to be seated in a row for a photograph . The students are to occupy the 4 chairs , two at each end and the lady does not wish to have student on either side . In how many ways the group can be seated **ans : 86400**

PERMUTATION - QSET 2

01. How many different arrangements can be made out of letters of the word " EQUATION" such that they begin with vowel and end with a consonant **ans: 10800**
02. How many different arrangements can be made out of letters of the word "TRIANGLE" such that they begin and end with a vowel **ans: 4320**
03. the number of arrangements in which letters of the word MONDAY be arranged so that the words thus formed begin with M and do not end with N is **ans: 96**
04. in how many ways can the letters of the word "STRANGLE" be arranged amongst themselves so that
- a) vowels occupy odd places **ans : 8640**
 - b) vowels are not together **ans : 30240**
 - c) vowels are always together **ans : 10800**
05. In how many ways can the letters of the word "FORMULA" be arranged amongst themselves so that
- a) consonants occupy the odd places **ans: 144**
 - b) vowels are always together **ans: 720**

06. In how many ways can the letters of the word "LOGARITHM" be arranged amongst themselves so that
- a) vowels are always together **ans: 30240**
 b) no two vowels are together **ans: 151200**
 c) consonants occupy even positions **ans : 43200**
 d) begin with 'O' and end with 'T' **ans : 5040**
07. In how many ways letters of the word "STORY" be arranged so that
- a) T and Y are always together **ans : 48**
 b) T is always next to Y **ans : 24**
08. the number of ways the letters of the word TRIANGLE to be arranged so that the word 'angle' will always be present **ans: 24**
09. in how many ways can the letters of the word "MOBILE" be arranged so that consonants occupy the old places **ans : 36**
- 10 for the word TRIANGLE , in how many arrangements the relative positions of the vowels and consonants remain unchanged **ans: 720**
- 11 For the word COMRADE , in how many arrangements the relative positions of the vowels and consonants remain unchanged **ans: 144**

PERMUTATION - QSET 3

01. a number of 4 different digits is formed by using the digits 1, 2, 3, 4, 5, 6, 7, 8 in all possible ways . Find how many numbers are greater than 3000 **ans : 1260**
02. a number of 4 different digits is to be formed by using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 . Find how many of them are a) greater than 4000 b) divisible by 2 c) divisible by 5
ans : 2016 ; 1344 , 336
03. How many 5 different digit numbers can be formed with digits 2, 3, 5, 7, 9 which are
 a) greater than 30000 b) less than 70000 c) between 30000 & 90000
ans : 96 , 72 , 72
04. How many 6 digit numbers can be formed from the digits 3, 4, 5, 6, 7, 8 if no digit is repeated . How many of these are a) divisible by 5 b) not divisible by 5
ans : 720 , 120 , 600

- 05.** how many different digit nos can be formed between 100 and 1000 using 0 , 1 , 3 , 5 and 7 which is not divisible by 5 **ans : 27**
- 06.** How many different digit nos are formed between 7000 and 8000 using 0 , 1 , 3 , 5 , 7 and 9 which are divisible by 5 **ans : 24**
- 07.** How many 5 – digit telephone numbers can be formed with digits 0 , 1 , 2 , , 9 if each numbers first 2 digits are 35 and no digit appears more than once . **ans : 336**
- 08.** A code word should consists of two English Capital alphabets followed by two distinct digits from 1 to 9 e.g. MH23 is a code word .
- a) how many such code words are available **ans : 46,800**
- b) how many of them end with an even integer **ans : 20,800**
- 09.** how many even numbers of four digits can be formed using digits 0 , 1 , 2 , 3 , 4 , 5 and 6 , no digit being used more than once **ans : 420**
- 10.** how many 5 different digit numbers can be formed with digits 0 , 1 , 3 , 5 , 6 , 8 and 9 divisible by 5 **ans : 660**

FUNDAMENTAL PRINCIPLE OF COUNTING

- 01.** There are 8 routes from city A to city B , 10 routes from city B to city C . Find the number of different ways for a person to travel from city A to city C via city B .

Person can travel from city A to city B by any one of the 8 routes in **8 ways**

Having done that ,

He can then travel from city B to city C by any one of the 10 routes in **10 ways**

By fundamental principle of Multiplication

Total ways in which the person can travel from city A to city C = $8 \times 10 = 80$

- 02.** a room has six doors . In how many ways can a man enter the room through one door and come out through a different door

man can enter the room through any one of the six doors in 6 ways

Having done that ,

He can then come out of the room through any one of the remaining 5 doors in 5 ways

By fundamental principle of Multiplication

Total ways in which a man can enter the room through one door and come out through a different door = $6 \times 5 = 30$

- 03.** There are 10 trains plying between Calcutta and Delhi . The number of ways in which a person can go from Calcutta to Delhi and return by a different train

person can go from Calcutta to Delhi by any one of the 10 trains in 10 ways

having done that ,

he can then return by any one of the remaining 9 trains in 9 ways

By fundamental principle of Multiplication

Total ways in which a person can go from Calcutta to Delhi and return by a different train = $10 \times 9 = 90$

- 04.** in how many ways can 3 prizes be distributed among 4 boys when no boy gets more than one prize .

1st prize can be distributed to any one of the 4 boys in 4 ways

Having done that ,

2nd prize can then be distributed to any one of the remaining 3 boys in 3 ways

Having done that ,

3rd prize can then be distributed to any one of the remaining 2 boys in 2 ways

By fundamental principle of Multiplication

Total ways in which 3 prizes be distributed among 4 boys = $4 \times 3 \times 2 = 24$

- 05.** in how many ways can 4 prizes be distributed among 3 boys if a boy can get any number of prizes

each of the 4 prizes can be distributed to any one of 3 boys in 3 ways

By fundamental principle of Multiplication

$$\text{Total ways in which 4 prizes be distributed among 3 boys} = 3 \times 3 \times 3 \times 3 = 81$$

- 06.** in how many ways can 3 students be associated with 4 chartered accountants if each chartered accountant can have at most 1 student

1 student can be associated to any one of the 4 chartered accountants in 4 ways

Having done that ;

2nd student can be associated to any one of the remaining 3 CA's in 3 ways

Having done that ;

3rd student can be associated to any one of the remaining 2 CA's in 2 ways

By fundamental principle of Multiplication

Total ways in which 3 students can be associated with 4 chartered accountants

$$= 4 \times 3 \times 2 = 24$$

- 07.** A letter lock contains 3 rings , each ring contains 5 different letters . Determine the maximum number of false trials that can be made before lock is opened

each ring can be set to any one of the 5 different letters in 5 ways

By fundamental principle of Multiplication

$$\text{Total ways in which the letter lock can be set} = 5 \times 5 \times 5 = 125$$

Out of all 125 combinations , there will be 1 combination in which the lock will be opened

. Hence maximum number of false trials that can be made before the lock is opened = 124

- 08.** For a set of 5 true/False questions no student has written all correct answers and no two students have given the same sequence of answers . What is the maximum number of students in the class for this to be possible

each true/False question can be answered in 2 ways

By fundamental principle of Multiplication

$$\text{Total ways in which student can answer 5 true/False questions} = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

Since no two students have given the same sequence of answers the maximum students in the class = 32

Also since no student has written all correct answers , the maximum students in the class will be = 32 - 1 = 31

09. Given 4 flags of different color , how many different signals can be generated if a signal requires to use 2 flags one below the other .

1st flag can be selected from any one of the 4 different color flags in 4 ways .

having done that ;

2nd flag can then be selected from any one of the remaining 3 flags in 3 ways

By fundamental principle of Multiplication

Total ways in which different signals can be generated using 2 flags = $4 \times 3 = 12$

10. there are 6 items in column A and 6 items in column B . A student is asked to match each item in column A with an item in column B . How many possible , correct , incorrect answers are there to this question

1st item in column A can be matched with any one of the 6 items in column B in 6 ways

Having done that

2nd item in column A can then be matched with any one of the remaining 5 items in column B in 5 ways

Having done that

3rd item in column A can then be matched with any one of the remaining 4 items in column A in 4 ways and so on

By fundamental principle of Multiplication

Total ways of answering this question = $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

11. How many 3 digit numbers can be formed from the digits 0, 2, 4 , 5, 7 if the repetition of digits is not allowed.

Hundred's place can be filled by any one of the 4 digits (2 , 4 , 5 , 7) in 4 ways .

Having done that ;

Ten's place can then be filled by any one of the remaining 4 digits ('0' included) in 4 ways

Having done that ;

unit place can then be filled by any one of the remaining 3 digits in 3 ways

By fundamental principle of Multiplication

Total 3 – digit numbers formed = $4 \times 4 \times 3 = 48$

12. How many 3 digit even numbers can be formed from the digits 1 , 2 , 3 , 4 , 5 if the digits can be repeated

Unit place can be filled by any one of the digits 2 & 4 in 2 ways

Having done that ;

Tens & Hundred place can be filled by any one of the 5 digits (repetition allowed) in 5 ways each .

By fundamental principle of Multiplication

$$\text{Total 3 – digit numbers formed} = 2 \times 5 \times 5 = 50$$

13. How many numbers between 100 and 1000 are such that exactly one of the digits is 6

Case 1 : '6' in the unit place

Unit place can be filled by digit '6' in 1 way

Having done that ; Ten's place can be filled by any one of the remaining 9 digits ('6' excluded) in 9 ways

Having done that ; Hundred's place can be filled by any one of the remaining 8 digits ('0' & '6' excluded) in 8 ways

$$\therefore \text{Numbers formed in this case} = 1 \times 9 \times 8 = 72$$

Case 2 : '6' in the ten's place

Ten's place can be filled by digit '6' in 1 way

Having done that ; Unit's place can be filled by any one of the remaining 9 digits ('6' excluded) in 9 ways

Having done that ; Hundred's place can be filled by any one of the remaining 8 digits ('0' & '6' excluded) in 8 ways

$$\therefore \text{Numbers formed in this case} = 1 \times 9 \times 8 = 72$$

Case 3 : '6' in the Hundred's place

Hundred's place can be filled by digit '6' in 1 way

Having done that ;

Ten's & Unit's place can be filled by any one of the remaining 9 digits ('6' excluded) in 9 ways each

$$\therefore \text{Numbers formed in this case} = 1 \times 9 \times 9 = 81$$

By fundamental principle of ADDITION

$$\text{Total 3 – digit numbers formed} = 72 + 72 + 81 = 225$$

SOLUTION - Q SET 1

01. 3 Asiatics ; 2 Europeans and 1 American stand in a line for a photograph . Find the number of arrangements so that 3 Asiatics are together and so are 2 Europeans .

Consider 3 Asiatics as 1 set & 2 Europeans as 1 set .

1 set of Asiatics , 1 set of European & 1 American can be arranged in ${}^3P_3 = 3!$ ways

Having done that ;

3 asiatics can be arranged in ${}^3P_3 = 3!$ ways

Having done that

2 Europeans can be arranged in ${}^2P_2 = 2!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 3! \times 3! \times 2! \\ &= 6 \times 6 \times 2 = 72 \end{aligned}$$

02. There are 7 students of whom 2 are Americans , 2 Russians and 3 Indians . They have to stand in a line so that 2 Americans are always together and 3 Indians are always together . In how many ways can this be done

Consider 2 Americans as 1 set of student & 3 Indians as 1 set

Therefore 1 set of American , 1 set of Indians and 2 Russians can be arranged in ${}^4P_4 = 4!$ ways

Having done that ;

2 Americans can be arranged in ${}^2P_2 = 2!$ ways

Having done that

3 Indians can be arranged in ${}^3P_3 = 3!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 4! \times 2! \times 3! \\ &= 24 \times 2 \times 6 = 188 \end{aligned}$$

03. 6 boys and 2 girls are to be stand in a line for a photograph . Find the number of arrangements such that

a) two girls sit next to each other b) two girls occupy the end seats

a) two girls sit next to each other

Consider 2 girls as 1 set

1 set of 2 girls & 6 boys can be arranged in ${}^7P_7 = 7!$ Ways

Having done that ;

The two girls can then be arranged among themselves in ${}^2P_2 = 2!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 7! \times 2! \\ &= 5040 \times 2 = 10080 \end{aligned}$$

b) two girls occupy the end seats

the two girls can be arranged on to the end seats in ${}^2P_2 = 2!$ Ways

Having done that ;

The 6 boys can then be arranged in ${}^6P_6 = 6!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 2! \times 6! \\ &= 2 \times 720 = 1440 \end{aligned}$$

- 04.** there are 4 books on Physics and 2 on Maths . Find the number of ways in which the books can be arranged so that books on Mathematics are not together

* P * P * P * P *

two math books can be arranged into any 2 of the 5 positions marked '*' in 5P_2 ways

Having done that ;

4 Physics books can then be arranged in ${}^4P_4 = 4!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= {}^5P_2 \times 4! \\ &= 20 \times 24 \\ &= 480 \end{aligned}$$

- 05.** Five persons are to be seated in a row . Find the number of seating arrangements such that
- a) if two persons X and Y are always together
 - b) if two person X and Y never sit together
 - c) if X and Y occupy chairs at two ends

a) two persons X and Y are always together

Consider X & Y as 1 set

1 set of X & Y & 3 others can be arranged in ${}^4P_4 = 4!$ ways

Having done that ;

X & Y can then be arranged among themselves in ${}^2P_2 = 2!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 4! \times 2! \\ &= 24 \times 2 \\ &= 48 \end{aligned}$$

b) **two person X and Y never sit together**

* ○ * ○ * ○ *

two persons X and Y can be arranged into any 2 of the 4 positions marked '*' in 4P_2 ways

Having done that ;

3 others can then be arranged in ${}^3P_3 = 3!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= {}^4P_2 \times 3! \\ &= 12 \times 6 = 72 \end{aligned}$$

c) **if X and Y occupy chairs at two ends**

X and Y can be arranged in to the end seats in ${}^2P_2 = 2!$ ways

Having done that ;

3 others can then be arranged in ${}^3P_3 = 3!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 2! \times 3! \\ &= 2 \times 6 = 12 \end{aligned}$$

06. In how many ways can 6 papers of which 2 are math can be arranged such that

- a) two math paper are consecutive b) the two math paper are not consecutive

a) **two math paper are consecutive**

Consider 2 math papers as 1 set

1 set of 2 math papers & 4 other papers can be arranged in

$${}^5P_5 = 5! \text{ ways}$$

Having done that ;

2 math papers can then be arranged among themselves in ${}^2P_2 = 2!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 5! \times 2! \\ &= 120 \times 2 = 240 \end{aligned}$$

b) **the two math paper are not consecutive**

* ○ * ○ * ○ * ○ *

2 math papers can be arranged into any 2 of the 5 positions marked '*' in ${}^5P_2 = 20$ ways

Having done that ;

4 other papers can then be arranged in ${}^4P_4 = 4!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= {}^5P_2 \times 4! \\ &= 20 \times 24 = 480 \end{aligned}$$

07. A family of 3 sisters and 5 brothers is to be arranged for a photograph in one row. In how many ways can they be seated if
- a) all the sisters sit together b) no two sisters sit together

a) all the sisters sit together

Consider 3 sisters as 1 set

1 set of 3 sisters & 5 Brothers can be arranged in ${}^6P_6 = 6!$ ways

Having done that ;

3 sisters can then be arranged among themselves in ${}^3P_3 = 3!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 6! \times 3! \\ &= 720 \times 6 = 4320 \end{aligned}$$

b) no two sisters sit together

* B * B * B * B * B *

3 sisters can be arranged into any 3 of the 6 positions marked '*' in 6P_3 ways

Having done that ;

5 Brothers can then be arranged in ${}^5P_5 = 5!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= {}^6P_3 \times 5! \\ &= 120 \times 120 = 14400 \end{aligned}$$

08. 7 persons sit in a row. Find the total number of seating arrangements if

- a) 3 persons A, B, C sit together in particular order
 b) A, B and C sit together in any order
 c) A and B occupy the end seats
 d) C always occupies the end seats

a) 3 persons A, B, C sit together in particular order

Consider A, B, C as 1 set

1 set of A, B, C & 4 others can be arranged in ${}^5P_5 = 5!$ ways

Having done that ;

Since A, B, C sit together in a particular order, they cannot be further arranged

By fundamental principle of Multiplication

$$\text{Total arrangements} = 5! = 120$$

b) A , B and C sit together in any order

Consider A , B , C as 1 set

1 set of A , B , C & 4 others can be arranged in ${}^5P_5 = 5!$ ways

Having done that ;

A, B , C can then be arranged among themselves in ${}^3P_3 = 3!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 5! \times 3! \\ &= 120 \times 6 = 720 \end{aligned}$$

c) A and B occupy the end seats

A and B can be arranged onto the end seats in ${}^2P_2 = 2!$ ways

Having done that ;

5 others can then be arranged among themselves in ${}^5P_5 = 5!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 2! \times 5! \\ &= 2 \times 120 = 240 \end{aligned}$$

d) C always occupies the end seats

C can be arranged into any one of the end seats in ${}^2P_1 = 2$ ways

Having done that ;

6 others can then be arranged among themselves in ${}^6P_6 = 6!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 2! \times 6! \\ &= 2 \times 720 = 1440 \end{aligned}$$

09. the college day committee consists of a Principal , four professors and two students . They are seated in a row for a photograph so that the Principal is in the center and the two students occupy the chairs at the ends . how many different photographs can be taken

ARRANGEMENT

Student Prof. Prof. Principal Prof. Prof. Student

Principal can be arranged onto the center seat in 1 way

Having done that ;

2 students can be arranged onto the end seats in ${}^2P_2 = 2!$ ways

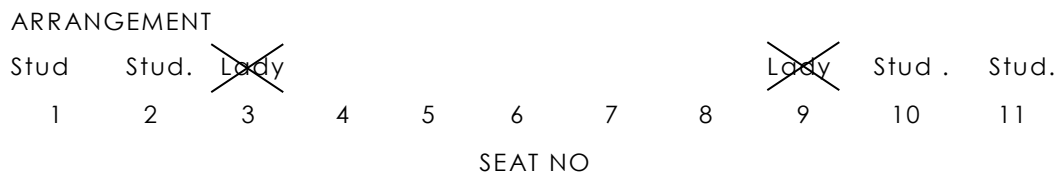
Having done that ;

The 4 professors can then be arranged into the remaining seats in ${}^4P_4 = 4!$ ways

By fundamental principle of Multiplication

$$\text{Total arrangements} = 2! \times 4! = 2 \times 24 = 48$$

10. 6 professors , 1 lady and 4 students are to be seated in a row for a photograph . The students are to occupy the 4 chairs , two at each end and the lady does not wish to have student on either side . In how many ways the group can be seated



4 students can be arranged into seats marked 1 , 2 ,10 & 11 in ${}^4P_4 = 4!$ ways

Having done that ;

Since the lady does not wish to have student on either side , she has to be arranged into any one of the seats marked 4 , 5 , 6 , 7 & 8 .

This can be done in ${}^5P_1 = 5$ ways

Having done that

the 6 professors can then be arranged into the remaining seats in ${}^6P_6 = 6!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 4! \times 5 \times 6! \\ &= 24 \times 5 \times 720 = 86400 \end{aligned}$$

SOLUTION - Q SET 2

01. How many different arrangements can be made out of letters of the word " EQUATION" such that they begin with vowel and end with a consonant .

' EQUATION '

8 – LETTER WORD

VOWELS : - A , E , I , O , U

CONSONANTS : - Q , T , N

1st place can be filled by any one of the 5 vowels in 5 ways

last place can be filled by any one of the 3 consonants in 3 ways

Having done that ;

Remaining 6 places can be filled by remaining 6 letters in ${}^6P_6 = 6!$ ways

By fundamental principle of Multiplication

$$\begin{aligned}\text{Total arrangements} &= 5 \times 3 \times 6! \\ &= 15 \times 720 &= 10800\end{aligned}$$

02. How many different arrangements can be made out of letters of the word "TRIANGLE" such that they begin and end with a vowel

' TRIANGLE '

8 – LETTER WORD

VOWELS : - I , A , E

CONSONANTS : - T , R , N , G , L

1st & last place can be filled by any 2 of the 3 vowels in ${}^3P_2 = 6$ ways

Having done that ;

Remaining 6 places can be filled by remaining 6 letters in ${}^6P_6 = 6!$ ways

By fundamental principle of Multiplication

$$\begin{aligned}\text{Total arrangements} &= 6 \times 6! \\ &= 6 \times 720 \\ &= 4320\end{aligned}$$

03. the number of arrangements in which letters of the word MONDAY be arranged so that the words thus formed begin with M and do not end with N

'MONDAY' - 6 - letter word

1ST place can be filled by letter 'M' in 1 way

Since the word must not end with 'N' ;

Last place can be filled by any one of the 4 letters O , D , A , Y in 4 ways

Having done that ,

Remaining 4 places can be filled by remaining 4 letters in ${}^4P_4 = 4!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 4 \times 4! \\ &= 4 \times 24 = 96 \end{aligned}$$

04. in how many ways can the letters of the word "STRANGLE" be arranged amongst themselves so that a) vowels occupy odd places b) vowels are not together c) vowels are together

'STRANGLE ' : 8 - LETTER WORD

VOWELS : - A , E ; CONSONANTS : - S , T , R , N , G , L

a) vowels occupy odd places

2 vowels can be arranged into any 2 of the 4 odd places in ${}^4P_2 = 12$ ways

having done that ;

the remaining 6 places can be filled by remaining 6 letters in ${}^6P_6 = 6!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 12 \times 6! \\ &= 12 \times 720 = 8640 \end{aligned}$$

b) vowels are not together

* C * C * C * C * C * C *

two vowels can be arranged into any 2 of the 7 positions marked '*' in 7P_2 ways

Having done that ;

the remaining 6 places can be filled by remaining 6 letters in ${}^6P_6 = 6!$ ways

By fundamental principle of Multiplication

$$\text{Total arrangements} = 42 \times 6! = 42 \times 720 = 30240$$

c) vowels are together

consider 2 vowels as 1 set .

set of 2 vowels & 6 consonants can be arranged in ${}^7P_7 = 7!$ Ways

Having done that ;

The 2 vowels can then be arranged amongst themselves in ${}^2P_2 = 2!$ ways

By fundamental principle of Multiplication

$$\begin{aligned}\text{Total arrangements} &= 7! \times 2! \\ &= 5040 \times 2 \\ &= 10800\end{aligned}$$

- 05.** In how many ways can the letters of the word "FORMULA" be arranged amongst themselves so that a) consonants occupy the odd places b) vowels are always together

'FORMULA' : 7 - LETTER WORD

VOWELS : - O , A , U ; CONSONANTS : - F , R , M , L

a) consonants occupy odd places

the 4 consonants can be arranged into 4 odd places ${}^4P_4 = 4!$ Ways

Having done that

the remaining 3 places can be filled by remaining 3 vowels in ${}^3P_3 = 3!$ ways

By fundamental principle of Multiplication

$$\begin{aligned}\text{Total arrangements} &= 4! \times 3! \\ &= 24 \times 6 = 144\end{aligned}$$

b) vowels are together

consider 3 vowels as 1 set .

1 set of 3 vowels & 4 consonants can be arranged in ${}^5P_5 = 5!$ Ways

Having done that ;

The 3 vowels can then be arranged amongst themselves in ${}^3P_3 = 3!$ ways

By fundamental principle of Multiplication

$$\begin{aligned}\text{Total arrangements} &= 5! \times 3! \\ &= 120 \times 6 \\ &= 720\end{aligned}$$

06. In how many ways can the letters of the word "LOGARITHM" be arranged amongst themselves so that a) no two vowels are together b) consonants occupy even positions c) begin with 'O' and end with 'T'

'LOGARITHM'

9 – LETTER WORD

VOWELS : - O , A , I ; CONSONANTS : - L , G , R , T , H , M

a) vowels are not together

* C * C * C * C * C * C *

3 vowels can be arranged into any 3 of the 7 positions marked '*' in ${}^7P_3 = 210$ ways

Having done that ;

the remaining 6 places can be filled by remaining 6 consonants in ${}^6P_6 = 6!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 210 \times 6! \\ &= 210 \times 720 = 151200 \end{aligned}$$

b) consonants occupy even places

the 4 out of 6 consonants can be arranged into the 4 even places in ${}^6P_4 = 360$ ways

Having done that

the remaining 5 places can be filled by remaining 5 letters in ${}^5P_5 = 5!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 360 \times 5! \\ &= 360 \times 120 = 43200 \end{aligned}$$

c) begin with 'O' and end with 'T'

1st place can be filled by letter 'O' in 1 way

Last place can be filled by letter 'T' in 1 way

Having done that ,

Remaining 7 places can be filled by remaining 7 letters in ${}^7P_7 = 7!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 1 \times 1 \times 7! \\ &= 5040 \end{aligned}$$

07. In how many ways letters of the word "STORY" be arranged so that

- a) T and Y are always together b) T is always next to Y

a) T and Y are always together

Consider T & Y as 1 set

1 set of T & Y & 3 other letters can be arranged in ${}^4P_4 = 4!$ ways

Having done that ;

The letters T & Y can then be arranged amongst themselves in ${}^2P_2 = 2!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 4! \times 2! \\ &= 24 \times 2 = 48 \end{aligned}$$

b) T is always next to Y

Consider T & Y as 1 set

1 set of T & Y & 3 other letters can be arranged in ${}^4P_4 = 4!$ ways

Since T is always next to Y , they **SHOULD NOT BE** be further arranged in ${}^2P_2 = 2!$ Ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 4! \\ &= 24 \end{aligned}$$

08. the number of ways the letters of the word TRIANGLE to be arranged so that the word 'angle' will always be present

Consider letters of the word 'ANGLE' as 1 set

\therefore 1 set of 'ANGLE' + T , R , I can be arranged in ${}^4P_4 = 4!$ Ways

Since word 'angle' has to be always present , the letters of the word 'angle' SHOULD NOT BE further arranged amongst themselves

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 4! \\ &= 24 \end{aligned}$$

09. in how many ways can the letters of the word "MOBILE" be arranged so that consonants occupy the old places

'MOBILE'

6 – LETTER WORD

VOWELS :- O , I , E ; CONSONANTS :- M , B , L

Since the consonants have to occupy the old places , they have to occupy position 1 , 3 & 5 and so they can be arranged in ${}^3P_3 = 3!$ ways

Having done that

the remaining 3 places can be filled by remaining 3 vowels in ${}^3P_3 = 3!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 3! \times 3! \\ &= 6 \times 6 = 36 \end{aligned}$$

10. For the word TRIANGLE , in how many arrangements the relative positions of the vowels and consonants remain unchanged

SOLUTIO

Letters	:	T	R	I	A	N	G	L	E
Vowel/Cons.	:	C	C	V	V	C	C	C	V
Position No.	:	1	2	3	4	5	6	7	8

Since the relative positions of vowels & consonants remain unchanged , the vowels have to be arranged into positions 3 , 4 & 8 AND the consonants have to arranged into positions 1 , 2 , 5 , 6 & 7 . Hence

Vowels can be arranged in ${}^3P_3 = 3!$ ways

Consonants can be arranged in ${}^5P_5 = 5!$ Ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 3! \times 5! \\ &= 6 \times 120 = 720 \end{aligned}$$

11. For the word COMRADE , in how many arrangements the relative positions of the vowels and consonants remain unchanged

Letters	:	C	O	M	R	A	D	E
Vowel/Cons.	:	C	V	C	C	V	C	V
Position No.	:	1	2	3	4	5	6	7

Since the relative positions of vowels & consonants remain unchanged , the vowels have to be arranged into positions 2 , 5 & 7 AND the consonants have to arranged into positions 1 , 3 , 4 , 6 . Hence

Vowels can be arranged in ${}^3P_3 = 3!$ ways

Consonants can be arranged in ${}^4P_4 = 4!$ Ways

By fundamental principle of Multiplication

$$\begin{aligned}
 \text{Total arrangements} &= 3! \times 4! \\
 &= 6 \times 24 = 144
 \end{aligned}$$

01. a number of 4 different digits is formed by using the digits 1, 2, 3, 4, 5, 6, 7, 8 in all possible ways. Find how many numbers are greater than 3000

thousand place can be filled by any one of the digits 3, 4, 5, 6, 7, 8 in 6P_1 ways

Having done that the remaining 3 places can be filled by any 3 of the remaining 7 digits in 7P_3 ways

By fundamental principle of Multipliation ,

$$\begin{aligned} \text{Total numbers formed} &= {}^6P_1 \times {}^7P_3 \\ &= 6 \times 7 \times 6 \times 5 = 1260 \end{aligned}$$

02. a number of 4 different digits is to be formed by using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9. Find how many of them are

a) greater than 4000

thousand place can be filled by any one of the digits 4, 5, 6, 7, 8, 9 in 6P_1 ways

Having done that the remaining 3 places can be filled by any 3 of the remaining 8 digits in 8P_3 ways

By fundamental principle of Multipliation ,

$$\begin{aligned} \text{Total numbers formed} &= {}^6P_1 \times {}^8P_3 \\ &= 6 \times 8 \times 7 \times 6 = 2016 \end{aligned}$$

b) divisible by 2

unit place can be filled by any one of the digits 2, 4, 6, 8 in 4P_1 ways

Having done that the remaining 3 places can be filled by any 3 of the remaining 8 digits in 8P_3 ways

By fundamental principle of Multipliation ,

$$\begin{aligned} \text{Total numbers formed} &= {}^4P_1 \times {}^8P_3 \\ &= 4 \times 8 \times 7 \times 6 = 1344 \end{aligned}$$

c) divisible by 5

unit place can be filled by digit '5' in 1 way

Having done that the remaining 3 places can be filled by any 3 of the remaining 8 digits in 8P_3 ways

By fundamental principle of Multipliation ,

$$\begin{aligned} \text{Total numbers formed} &= 1 \times {}^8P_3 \\ &= 8 \times 7 \times 6 = 336 \end{aligned}$$

03. How many 5 different digit numbers can be formed with digits 2, 3, 5, 7, 9 which are

a) greater than 30000

thousand place can be filled by any one of the digits 3, 5, 7, 9 in 4P_1 ways

Having done that the remaining 4 places can be filled by remaining 4 digits in ${}^4P_4 = 4!$ ways

By fundamental principle of Multiplication,

$$\text{Total numbers formed} = {}^4P_1 \times 4! = 4 \times 24 = 96$$

b) less than 70000

thousand place can be filled by any one of the digits 2, 3, 5 in 3P_1 ways

Having done that the remaining 4 places can be filled by remaining 4 digits in ${}^4P_4 = 4!$ ways

By fundamental principle of Multiplication,

$$\text{Total numbers formed} = {}^3P_1 \times 4! = 3 \times 24 = 72$$

b) between 30000 & 90000

thousand place can be filled by any one of the digits 3, 5, 7 in 3P_1 ways

Having done that the remaining 4 places can be filled by remaining 4 digits in ${}^4P_4 = 4!$ ways

By fundamental principle of Multiplication,

$$\text{Total numbers formed} = {}^3P_1 \times 4! = 3 \times 24 = 72$$

04. How many 6 digit numbers can be formed from the digits 3, 4, 5, 6, 7, 8 if no digit is repeated. How many of these are

a) divisible by 5

unit place can be filled by digit '5' in 1 way

Having done that,

Remaining 5 places can be filled by the remaining 5 digits in ${}^5P_5 = 5!$ ways

By fundamental principle of Multiplication

$$\text{Total numbers formed} = 5! = 120$$

b) not divisible by 5

unit place can be filled by any one of the 5 digits ('5' excluded) in 5 ways

Having done that,

Remaining 5 places can be filled by the remaining 5 digits in ${}^5P_5 = 5!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total numbers formed} &= 5 \times 5! \\ &= 5 \times 120 \\ &= 600 \end{aligned}$$

- 05.** how many different digit numbers can be formed between 100 and 1000 using 0 , 1 , 3 , 5 and 7 which is not divisible by 5

unit place can be filled by any one of digits 1 , 3 & 7 in 3P_1 ways

Having done that ,

Hundreds place can be filled by any one the remaining 3 digits ('0' excluded) in 3P_1 ways

Having done that , tens place can then be filled by any one of the remaining 3 digits in 3P_1 ways

By fundamental principle of Multipliation ,

$$\text{Total numbers formed} = {}^3P_1 \times {}^3P_1 \times {}^3P_1 = 3 \times 3 \times 3 = 27$$

- 06.** How many different digit nos are formed between 7000 and 8000 using 0 , 1 , 3 , 5 , 7 and 9 which are divisible by 5

thousand place can be filled by digit '7' in 1 way

Having done that , units place can be filled by any one of the digits 0 , 5 in 2P_1 ways

Having done that ,

remaining 2 places can be filled by any 2 of the remaining 4 digits in 4P_2 ways

By fundamental principle of Multipliation ,

$$\begin{aligned} \text{Total numbers formed} &= 1 \times {}^2P_1 \times {}^4P_2 \\ &= 1 \times 2 \times 4 \times 3 = 24 \end{aligned}$$

- 07.** How many 5 – digit telephone numbers can be formed with digits 0 , 1 , 2 , , 9 if each numbers first 2 digits are 35 and no digit appears more than once .

First two places can be filled by digits '3' & '5' respectively in 1 way .

Having done that ,

The remaining 3 places can be filled by any 3 of the remaining 8 digits in 8P_3 ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total numbers formed} &= {}^8P_3 \\ &= 8 \cdot 7 \cdot 6 \\ &= 336 \end{aligned}$$

- 08.** A code word should consists of two English Capital alphabets followed by two distinct digits from 1 to 9 e.g. MH23 is a code word .

a) how many such code words are available

first 2 places can be filled by any 2 of the 26 alphabets in ${}^{26}P_2$ ways

Having done that ;

The last two places can be filled by 2 of the 9 digits in 9P_2 ways

By fundamental principle of Multiplication

$$\begin{aligned}\text{Total numbers formed} &= {}^{26}P_2 \times {}^9P_2 \\ &= 26 \times 25 \times 9 \times 8 = 46,800\end{aligned}$$

b) how many of them end with an even integer

first 2 places can be filled by any 2 of the 26 alphabets in ${}^{26}P_2$ ways

Having done that ;

since the code has to end with an even integer ,

The last (fourth) place can be filled by one of the digits 2 , 4 , 6 , 8 in 4 ways

Having done that ,

the third place can then be filled by any of the remaining 8 digits in 8 ways

By fundamental principle of Multiplication

$$\begin{aligned}\text{Total numbers formed} &= {}^{26}P_2 \times 4 \times 8 \\ &= 26 \times 25 \times 4 \times 8 \\ &= 20,800\end{aligned}$$

09. how many even numbers of four digits can be formed using digits 0 , 1 , 2 , 3 , 4 , 5 and 6 , no digit being used more than once

Step 1 : 4 – digit numbers formed

Thousand place can be filled by any one of the 6 digits ('0' excluded) in 6 ways

Having done that , remaining 3 places can be filled by any of remaining 6 digits in 6P_3 ways

By fundamental principle of Multipliation ,

$$\text{Numbers formed} = 6 \times {}^6P_3 = 6 \times 6 \times 5 \times 4 = 720$$

Step 2 : Odd Numbers

Unit place can be filled by any one of digits 1 , 3 , 5 in 3P_1 ways

Having done that ,

Thousand place can be filled by any one the remaining 5 digits ('0' excluded) in 5P_1 ways

Having done that , remaining 2 places can be filled by any of remaining 5 digits in 5P_2 ways

By fundamental principle of Multipliation ,

$$\text{No. of Odd Numbers formed} = {}^3P_1 \times {}^5P_1 \times {}^5P_2 = 3 \times 5 \times 5 \times 4 = 300$$

Hence

$$\text{No. of Even numbers formed} = 720 - 300 = 420$$

10. how many 5 different digit numbers can be formed with digits 0 , 1 , 3 , 5 , 6 , 8 and 9 divisible by 5

Step 1 : 5 – digit numbers formed

Ten Thousand place can be filled by any one of the 6 digits ('0' excluded) in 6 ways

Having done that , remaining 4 places can be filled by any of remaining 6 digits in 6P_4 ways

By fundamental principle of Multipliation ,

$$\text{Numbers formed} = 6 \times {}^6P_4 = 6 \times 6 \times 5 \times 4 \times 3 = 2160$$

Step 2 : Numbers not divisible by '5'

Unit place can be filled by any one of digits 1 , 3 , 6 , 8 , 9 in 5P_1 ways

Having done that ,

Ten Thousand place can be filled by any one the remaining 5 digits ('0' excluded) in 5P_1 ways

Having done that the remaining 3 places can be filled by any of remaining 5 digits in 5P_3 ways

By fundamental principle of Multipliation ,

$$\text{Numbers formed not divisible by 5} = {}^5P_1 \times {}^5P_1 \times {}^5P_3 = 5 \times 5 \times 5 \times 4 \times 3 = 1500$$

Hence

$$\text{Numbers divisible by 5} = 2160 - 1500 = 660$$

PERMUTATION - Q SET 4

01. $(n + 1)! = 42(n - 1)!$. Find n

02. $(n + 3)! = 110(n + 1)!$. Find n

03. $\frac{(16 - n)!}{(14 - n)!} = 12$
Find n

04. $\frac{n!}{3!(n - 3)!} : \frac{n!}{5!(n - 5)!} = 5 : 3$

05. $\frac{n!}{3!(n - 5)!} : \frac{n!}{5!(n - 7)!} = 10 : 3$

06. $\frac{2n!}{3!(2n - 3)!} : \frac{n!}{2!(n - 2)!} = 12 : 1$

07. ${}^8P_5 = {}^7P_5 + k \cdot {}^7P_4$, find k

08. ${}^{15}P_8 = 8 \cdot {}^{14}P_7 + {}^{14}P_r$, find r

09. ${}^nP_4 = 12({}^nP_2)$. Find n

10. ${}^nP_3 : {}^nP_6 = 1 : 210$. Find n

11. ${}^{n+2}P_4 : {}^{n+3}P_6 = 1 : 14$. Find n

12. ${}^nP_3 : {}^{n-1}P_3 = 5 : 4$. Find n

13. ${}^{11}P_{(r-1)} : {}^{12}P_{(r-2)} = 14 : 3$. Find r

14. if ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$

15. ${}^{x+y}P_2 = 56$ & ${}^{x-y}P_2 = 12$. Find x & y

01. $(n + 1)! = 42(n - 1)!$. Find n

$$(n + 1)! = 42(n - 1)!$$

$$\frac{(n + 1)!}{(n - 1)!} = 42$$

$$\frac{(n + 1) \cdot \cancel{n} \cdot \cancel{(n - 1)!}}{\cancel{(n - 1)!}} = 42$$

$$(n + 1) \cdot n = 42$$

On compare : $n = 6$

02. $(n + 3)! = 110(n + 1)!$. Find n

$$(n + 3)! = 110(n + 1)!$$

$$\frac{(n + 3)!}{(n + 1)!} = 110$$

$$\frac{(n + 3) \cdot (n + 2) \cdot \cancel{(n + 1)!}}{\cancel{(n + 1)!}} = 110$$

$$(n + 3) \cdot (n + 2) = 110$$

On compare : $n + 3 = 11$
 $n = 8$

03. $\frac{(16 - n)!}{(14 - n)!} = 12$
Find n

$$\frac{(16 - n)!}{(14 - n)!} = 12$$

$$\frac{(16 - n) \cdot (15 - n) \cdot \cancel{(14 - n)!}}{\cancel{(14 - n)!}} = 12$$

$$(16 - n) \cdot (15 - n) = 12$$

On compare : $16 - n = 4$
 $n = 12$

$$04. \frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-5)!} = 5 : 3$$

$$\frac{\frac{n!}{3!(n-3)!}}{\frac{n!}{5!(n-5)!}} = \frac{5}{3}$$

$$\frac{5!(n-5)!}{3!(n-3)!} = \frac{5}{3}$$

$$\frac{5.4 \cdot \cancel{3!} \cdot \frac{(n-5)!}{(n-3)(n-4)(n-5)!}}{\cancel{3!}} = \frac{5}{3}$$

$$\frac{5.4}{(n-3)(n-4)} = \frac{5}{3}$$

$$\frac{4}{(n-3)(n-4)} = \frac{1}{3}$$

$$(n-3)(n-4) = 4.3$$

$$\text{On compare : } \begin{aligned} n-3 &= 4 \\ n &= 7 \end{aligned}$$

$$05. \frac{n!}{3!(n-5)!} : \frac{n!}{5!(n-7)!} = 10 : 3$$

$$\frac{\frac{n!}{3!(n-5)!}}{\frac{n!}{5!(n-7)!}} = \frac{10}{3}$$

$$\frac{5!(n-7)!}{3!(n-5)!} = \frac{10}{3}$$

$$\frac{5.4 \cdot \cancel{3!} \cdot \frac{(n-7)!}{(n-5)(n-6)(n-7)!}}{\cancel{3!}} = \frac{10}{3}$$

$$\frac{5.4}{(n-5)(n-6)} = \frac{10}{3}$$

$$\frac{2}{(n-5)(n-6)} = \frac{1}{3}$$

$$(n-5)(n-6) = 3.2$$

$$\text{On compare : } \begin{aligned} n-5 &= 3 \\ n &= 8 \end{aligned}$$

$$06. \frac{2n!}{3!(2n-3)!} : \frac{n!}{2!(n-2)!} = 12 : 1$$

$$\frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{2!(n-2)!}} = \frac{12}{1}$$

$$\frac{2n!}{3!(2n-3)!} \times \frac{2!(n-2)!}{n!} = \frac{12}{1}$$

$$\frac{2n.(2n-1)(2n-2)(2n-3)! \times \frac{2!(n-2)!}{n(n-1)(n-2)!}}{\cancel{3!(2n-3)!}} = 12$$

$$\frac{2n.(2n-1)(2n-2)}{\cancel{3.2!}} \times \frac{\cancel{2!}}{n(n-1)} = 12$$

$$\frac{2n.(2n-1)(2n-2)}{3.n(n-1)} = 12$$

$$\frac{(2n-1)(2n-2)}{(n-1)} = 18$$

$$\frac{(2n-1)2.(n-1)}{\cancel{(n-1)}} = 18$$

$$2n-1 = 9 \quad \therefore n = 5$$

$$07. \quad 8 P_5 = 7 P_5 + k. 7 P_4, \text{ find } k$$

$$\frac{8!}{(8-5)!} = \frac{7!}{(7-5)!} + k \frac{7!}{(7-4)!}$$

$$\frac{8!}{3!} = \frac{7!}{2!} + k \frac{7!}{3!}$$

$$\frac{8.7!}{3.2!} = \frac{7!}{2!} + k \frac{7!}{3.2!}$$

$$\frac{8}{3} = 1 + \frac{k}{3}$$

$$\frac{8}{3} = \frac{3+k}{3}$$

$$8 = 3+k \quad k = 5$$

08.

${}^{15}P_8 = 8 \cdot {}^{14}P_7 + 14 P_r$, find r

$$\frac{15!}{(15-8)!} = 8 \frac{14!}{(14-7)!} + \frac{14!}{(14-r)!}$$

$$\frac{15 \cdot \cancel{14!}}{7!} = 8 \frac{\cancel{14!}}{7!} + \frac{\cancel{14!}}{(14-r)!}$$

$$\frac{15}{7!} = \frac{8}{7!} + \frac{1}{(14-r)!}$$

$$\frac{15-8}{7!} = \frac{1}{(14-r)!}$$

$$\frac{15-8}{7!} = \frac{1}{(14-r)!}$$

$$\frac{7}{7!} = \frac{1}{(14-r)!}$$

$$\frac{1}{6!} = \frac{1}{(14-r)!}$$

$$(14-r)! = 6!$$

$$14-r = 6$$

$$r = 8$$

09. ${}^n P_4 = 12 ({}^n P_2)$

$$\frac{\cancel{n!}}{(n-4)!} = 12 \frac{\cancel{n!}}{(n-2)!}$$

$$\frac{(n-2)!}{(n-4)!} = 12$$

$$\frac{(n-2)(n-3)\cancel{(n-4)!}}{\cancel{(n-4)!}} = 12$$

(DESCENDING ORDER)

$$(n-2)(n-3) = 4 \cdot 3$$

On comparing

$$n-2 = 4$$

$$n = 6$$

10. ${}^n P_3 : {}^n P_6 = 1 : 210$

$$\frac{{}^n P_3}{{}^n P_6} = \frac{1}{210}$$

$$\frac{\frac{n!}{(n-3)!}}{\frac{n!}{(n-6)!}} = \frac{1}{210}$$

$$\frac{(n-6)!}{(n-3)!} = \frac{1}{210}$$

$$\frac{(n-6)!}{(n-3)(n-4)(n-5)(n-6)!} = \frac{1}{210}$$

$$\frac{1}{(n-3)(n-4)(n-5)} = \frac{1}{210}$$

$$(n-3)(n-4)(n-5) = 210$$

$$(n-3)(n-4)(n-5) = 7 \cdot 6 \cdot 5$$

↑
 (DESCENDING ORDER)

2	210
3	105
5	35
7	7
	1

On Comparing

$$n-3 = 7$$

$$n = 10$$

11. ${}^{n+2} P_4 : {}^{n+3} P_6 = 1 : 14$

$$\frac{{}^{n+2} P_4}{{}^{n+3} P_6} = \frac{1}{14}$$

$$\frac{\frac{(n+2)!}{(n+2-4)!}}{\frac{(n+3)!}{(n+3-6)!}} = \frac{1}{14}$$

$$\frac{\frac{(n+2)!}{(n-2)!}}{\frac{(n+3)!}{(n-3)!}} = \frac{1}{14}$$

$$\frac{(n+2)!}{(n-2)!} \times \frac{(n-3)!}{(n+3)!} = \frac{1}{14}$$

$$\frac{(n+2)!}{(n+3)!} \times \frac{(n-3)!}{(n-2)!} = \frac{1}{14}$$

$$\frac{(n+2)!}{(n+3)(n+2)!} \times \frac{(n-3)!}{(n-2)(n-3)!} = \frac{1}{14}$$

$$\frac{1}{(n+3)(n-2)} = \frac{1}{14}$$

$$(n+3)(n-2) = 14$$

$$(n+3)(n-2) = 7 \cdot 2$$

On Comparing ;

$$n+3 = 7$$

$$n = 7 - 3 = 4$$

$$12. \quad {}^n P_3 : {}^{n-1} P_3 = 5 : 4$$

$$\frac{{}^n P_3}{{}^{n-1} P_3} = \frac{5}{4}$$

$$\frac{(n)!}{(n-3)!} = \frac{5}{4}$$

$$\frac{(n-1)!}{(n-1-3)!} = \frac{5}{4}$$

$$\frac{n!}{(n-3)!} = \frac{5}{4}$$

$$\frac{(n-1)!}{(n-4)!}$$

$$\frac{n!}{(n-3)!} \times \frac{(n-4)!}{(n-1)!} = \frac{5}{4}$$

$$\frac{n!}{(n-3)!} \times \frac{(n-4)!}{(n-1)!} = \frac{5}{4}$$

$$\frac{n!}{(n-1)!} \times \frac{(n-4)!}{(n-3)!} = \frac{5}{4}$$

$$\frac{n(n-1)!}{(n-1)!} \times \frac{(n-4)!}{(n-3)(n-4)} = \frac{5}{4}$$

$$\frac{n}{n-3} = \frac{5}{4}$$

$$4n = 5n - 15$$

$$n = 15$$

$$13. \quad {}^{11} P_{(r-1)} : {}^{12} P_{(r-2)} = 14 : 3$$

$$\frac{{}^{11} P_{r-1}}{{}^{12} P_{r-2}} = \frac{1}{14}$$

$$\frac{11!}{(11-r+1)!} = \frac{1}{14}$$

$$\frac{12!}{(12-r+2)!}$$

$$\frac{11!}{(12-r)!} = \frac{1}{14}$$

$$(14-r)!$$

$$\frac{11!}{(12-r)!} \times \frac{(14-r)!}{12!} = \frac{1}{14}$$

$$\frac{11!}{12!} \times \frac{(14-r)!}{(12-r)!} = \frac{1}{14}$$

$$\frac{11!}{12 \cdot 11!} \times \frac{(14-r)(13-r)(12-r)!}{(12-r)!} = \frac{1}{14}$$

$$\frac{1}{(n+3)(n-2)} = \frac{1}{14}$$

$$(n+3)(n-2) = 14$$

$$(n+3)(n-2) = 7 \cdot 2$$

On Comparing ;

$$n+3 = 7$$

$$n = 7 - 3 = 4$$

14. if ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$

$$\frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \frac{30800}{1}$$

$$\frac{\frac{56!}{(56-r-6)!}}{\frac{54!}{(54-r-3)!}} = 30800$$

$$\frac{\frac{56!}{(50-r)!}}{\frac{54!}{(51-r)!}} = 30800$$

$$\frac{56!}{(50-r)!} \times \frac{(51-r)!}{54!} = 30800$$

$$\frac{56!}{54!} \times \frac{(51-r)!}{(50-r)!} = 30800$$

$$\frac{56 \cdot 55 \cdot \cancel{54!}}{\cancel{54!}} \times \frac{(51-r)(50-\cancel{r})!}{(50-r)!} = 30800$$

$$56 \cdot 55 \cdot (51-r) = 30800$$

$$51-r = \frac{30800}{56 \cdot 55}$$

$$51-r = 10$$

$$r = 41$$

15. $x+y P_2 = 56$ & $x-y P_2 = 12$

$$x+y P_2 = 56$$

$$\frac{(x+y)!}{(x+y-2)!} = 56$$

$$\frac{(x+y)(x+y-1)(x+y-2)!}{(x+y-2)!} = 56$$

$$(x+y)(x+y-1) = 56$$

$$x+y = 8 \dots\dots\dots (1)$$

$$x-y P_2 = 12$$

$$\frac{(x-y)!}{(x-y-2)!} = 12$$

$$\frac{(x-y)(x-y-1)(x-y-2)!}{(x-y-2)!} = 12$$

$$(x-y)(x-y-1) = 12$$

$$x-y = 4 \dots\dots\dots (2)$$

solving (1) & (2)

$$x+y = 8$$

$$x-y = 4$$

$$2x = 12$$

$$x = 6$$

subs in (1)

$$y = 2$$